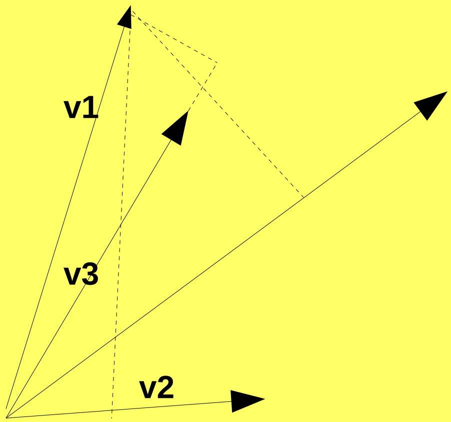
Breton: "So you have proven the proposition for all cases.

Einstein: "This is hard to visualize.

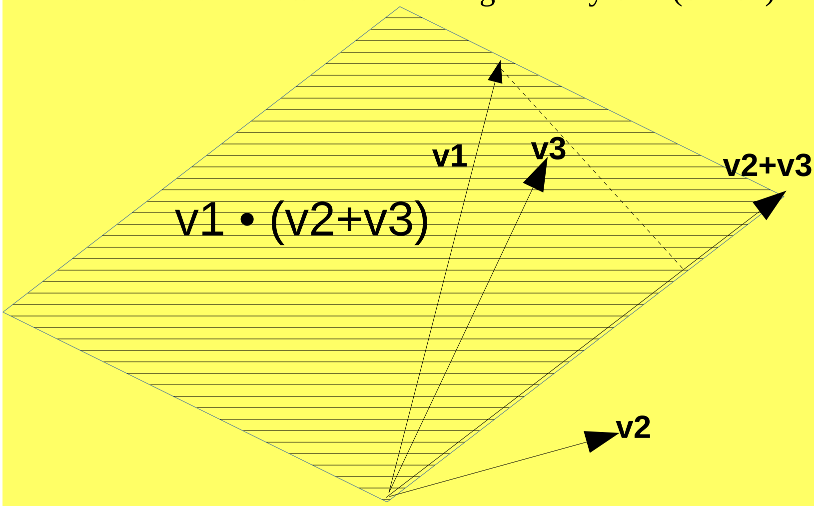
Breton: "We have proven the proposition algebraically, but the geometric rendition remains obscured. Some drawings may be helpful.

In a few minutes Breton handed his friends these three sketches.



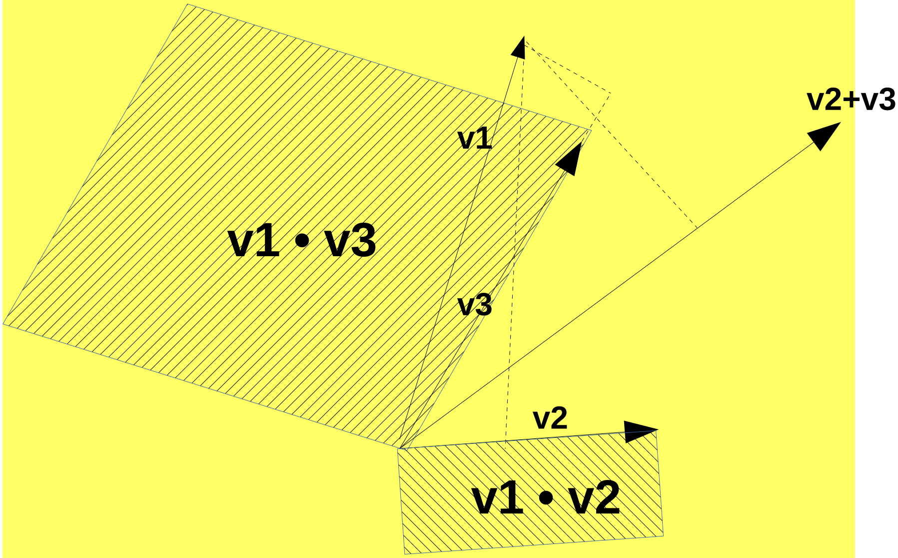
Breton: "This first sketch shows the two vectors, **v2** and **v3** and their sum lying in the same plane. The vector **v1** sticks up from the plane. The dotted lines show the orthogonals from from **v1** to **v2**, **v3**, and **v2+v3**. The orthogonals are related to inner products.

This next sketch show the area designated by **v1 • (v2+v3)**



Breton: "Geometrically, this area lies in a plane orthogonal to the plane defined by **v1** and (**v2+v3**).

The next sketch shows the two areas **v1 • v2** and **v1 • v3**.



Breton: "While the algebraic proof requires fine reasoning, the geometric proof would be even more difficult. Now are you convinced.

Einstein grudgingly: "It does follows that

$$\mathbf{v1} \bullet (\mathbf{v2}+\mathbf{v3}) = \mathbf{v1} \bullet \mathbf{v2} + \mathbf{v1} \bullet \mathbf{v3}$$

Breton, pressing the victory to a deliciously bitter ending: "The conclusion is ambiguous. If your mean

$$\mathbf{v1} \bullet (\mathbf{v2}+\mathbf{v3}) = \mathbf{v1} \bullet (\mathbf{v2} + \mathbf{v1}) \bullet \mathbf{v3}$$

then the result is a inner product between a scalar and a vector, which is meaningless. If you mean

$$\mathbf{v1} \bullet (\mathbf{v2}+\mathbf{v3}) = (\mathbf{v1} \bullet \mathbf{v2}) + (\mathbf{v1} \bullet \mathbf{v3})$$

then the result is the sum of two scalar quantities, a meaningful result."

After a short pause Breton continued in an agreeable tone. “Your reasoning follows the format for our formal proofs. Why not use the format we agreed upon? But before that, I suggest we simplify our notation. Let us write

q1 for q(**v1**)
q2 for q(**v2**)
q3 for q(**v3**)
uv1 for **u**(**v1**)
uv2 for **u**(**v2**)
uv3 for **u**(**v3**)

Whenever no ambiguity will follow, we can do the same in other contexts.

Einstein joining gladly: “Agreed. Here's my proof.”